

AD-A062 182

DESMATICS INC STATE COLLEGE PA

F/G 9/2

PERFORMANCE EVALUATION OF A SPECIFIC FACTOR SCREENING TECHNIQUE—ETC(U)

MAR 80 D E SMITH

N00014-79-C-0650

UNCLASSIFIED

TR-113-1

NL

1-1  
2-1



END

DATE

FILMED

4-80

DTIC

DESMATICS, INC.

(12)  
P. O. Box 618  
State College, Pa. 16801  
Phone: (814) 238-9621

*Applied Research in Statistics - Mathematics - Operations Research*

(3)  
PERFORMANCE EVALUATION OF A  
SPECIFIC FACTOR SCREENING TECHNIQUE.

by

(10) Dennis E./Smith

DTIC  
REPRODUCTION  
UNLIMITED

(14) TR-113-1

(1) TECHNICAL REPORT NO. 113-1

(12) 1231  
(11) March 1980  
(15)

This study was supported by the Office of Naval Research  
under Contract No. N00014-79-C-0650, Task No. NR 274-317  
and Contract No. N00014-75-C-1054, Task No. NR 042-334

Reproduction in whole or in part is permitted  
for any purpose of the United States Government

Approved for public release; distribution unlimited

34410-

### ABSTRACT

In many experimental situations (particularly in computer simulation studies) a large number of potentially important factors exist. Because of time and budget limitations, it is imperative to screen these factors in order to identify a subset which should be subjected to more detailed examination. This paper evaluates the performance of a factor screening technique which has been proposed for use when it is known that there is at most one active factor (i.e., a factor which has an effect on the response of interest). Performance evaluation reveals that the existence of even a relatively small amount of random error renders essentially useless a procedure which performs well in the deterministic case.

#### Key Words:

Factor screening

Experimental design

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DOC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	<input type="checkbox"/>
By _____	
Distribution/	
Availability Notes	
Date of Review	
Initial	

A

## TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT . . . . .	1
I. INTRODUCTION . . . . .	1
II. APPLICATION OF THE REVISED OTT-WEHRFRITZ PROCEDURE	3
A. ANALYSIS IN THE DETERMINISTIC CASE . . . . .	3
B. ANALYSIS IN THE NONDETERMINISTIC CASE . . . . .	5
III. PERFORMANCE EVALUATION . . . . .	9
A. MONTE CARLO PROCEDURES . . . . .	10
B. RESULTS . . . . .	11
IV. SUMMARY AND DISCUSSION . . . . .	16
V. REFERENCES . . . . .	17

## I. INTRODUCTION

In many experimental situations (particularly in computer simulation studies) a large number of potentially important factors exist. Because of time and budget limitations, it is imperative to screen these factors in order to identify a subset which should be subjected to more detailed examination. In general, the screening situation is one in which only a small number of the factors are actually active (i.e., have an effect on the response of interest). A number of factor screening approaches have been suggested. [See Kleijnen (1975) for a summary.]

This paper will concentrate on the situation in which it is known that there is at most one active factor in the set of factors to be screened. In a sense, the problem is somewhat analogous to searching for one possible needle in a haystack.

The first-order model

$$y = \beta_0 + \sum_{i=1}^K \beta_i x_i + \epsilon$$

will be assumed, where

- (1)  $y$  is the response of interest
- (2)  $x_i$  is a factor at two levels (+1 and -1)
- (3)  $\epsilon$  is a random error term with  $\epsilon \sim N(0, \sigma^2)$
- (4)  $K$  is the number of factors to be screened
- (5a)  $\beta_j = \Delta \neq 0$  and  $\beta_i = 0 (i \neq j)$  for unknown  $j$  if the  $j^{\text{th}}$  factor is active
- (5b)  $\beta_i = 0 (i = 1, \dots, K)$  if no factor is active.

For this situation Ott and Wehrfritz (1972) have developed a procedure for examining up to  $K = 2^N - 1$  factors in  $N$  runs under the additional assumptions that  $\sigma = 0$  (no random error) and  $\Delta > 0$  (direction of active factor effect known). Their procedure is concerned only with the selection of the active factor, and not with determining the value of  $\Delta$ .

This paper will concentrate on a slightly revised Ott-Wehrfritz procedure. Specifically, the performance of this revised procedure will be examined when the two assumptions  $\sigma = 0$  and  $\Delta > 0$  are not made. To compensate for the lack of information about the direction of active factor effect, an extra run will be required. Thus, this revised Ott-Wehrfritz procedure may be used to examine up to  $K = 2^N - 1 - 1$  factors in  $N$  runs.

Note the number of runs that can be saved by using this procedure. For example, a total of 127 factors can be screened using only eight runs. Of course, a savings in number of runs is truly a savings only if the procedure performs well. An evaluation of that performance is the main topic of this paper.

## II. APPLICATION OF THE REVISED OTT-WEHRFRITZ PROCEDURE

An easy method of generating an N-run screening design based on the Ott-Wehrfritz procedure when not assuming  $\Delta > 0$  is given in the following steps:

- (G1) Write (to the same number of places) the first  $2^N - 1 - 1$  positive binary numbers in ascending order. Include leading zeros so that each number has at least one leading zero.
- (G2) Replace the zeros by -1's.
- (G3) Let the resulting set of +1's and -1's corresponding to each binary number constitute a column in the design matrix, keeping the same order.

As an example, consider the case  $N = 4$ . Corresponding steps G1 through G3 and the resulting screening design are indicated in Figure 1. It should be noted that this design is the same one given by Ott and Wehrfritz (1972), but with permuted columns and an additional first row of -1's (because of the lack of the assumption that  $\Delta > 0$ ).

### A. ANALYSIS IN THE DETERMINISTIC CASE

In the deterministic case ( $\sigma = 0$ ), the analysis may be summarized in the following four steps:

- (D1) Observe the N responses  $y_1, \dots, y_N$

$$N = 4, 2^{N-1} - 1 = 7$$

[illegible]

Figure 1: Generation of a Screening Design for the Revised Ott-Wehrfritz Procedure When  $N = 4$



(D2) Define, for  $i = 1, \dots, N$ ,

$$w_i = \begin{cases} 0 & \text{if } y_i = y_1 \\ 1 & \text{if } y_i \neq y_1 \end{cases}$$

(D3) Calculate the number  $L = \frac{\sum_{i=1}^N (2^i - 2) w_i}{2}$

(D4) If  $L = 0$ , conclude that there is no active factor. Otherwise, select the  $L^{\text{th}}$  factor as the active factor.

Suppose that for the example considered, the observed responses were  $y_1 = 4.0$ ,  $y_2 = -2.0$ ,  $y_3 = 4.0$ , and  $y_4 = -2.0$  (step D1). Step D2 produces  $w_2 = 1$ ,  $w_3 = 0$ , and  $w_4 = 1$ , while steps D3 and D4 result in the selection of factor #5 as the active factor. Further, the first-order model is given by

$$y = 1.0 - 3.0 x_5$$

## B. ANALYSIS IN THE NONDETERMINISTIC CASE

In the deterministic case, the observed  $y_i$ 's assume at most two values. In the nondeterministic case ( $\sigma > 0$ ), however, all of the  $y_i$ 's assume different values with probability one. Because of this, analysis steps D1 through D4 for the deterministic case cannot be used. Instead, the analysis can be based on the statistic BSS/WSS, the ratio of the between sum of squares to the within sum of squares. The corresponding analysis steps are:

(N1) Order the observed  $y_i$ 's as  $y_{(1)} < y_{(2)} < \dots < y_{(N)}$

(N2) Consider  $N - 1$  partitions of these observations into two

groups, with the  $m^{\text{th}}$  partition ( $m = 1, \dots, N - 1$ ) consisting of:

Group 1:  $y_{(1)}, \dots, y_{(m)}$

Group 2:  $y_{(m+1)}, \dots, y_{(N)}$

(N3) For each partition let  $\bar{y}_{G1}$  denote the average of the first group and  $\bar{y}_{G2}$  denote the average of the second group. Define the ratio  $R_m = \text{BSS}_m / \text{WSS}_m$ , where

$$\text{BSS}_m = m(N - m) (\bar{y}_{G1} - \bar{y}_{G2})^2 / N$$

$$\text{WSS}_m = \sum_{i=1}^m (y_{(i)} - \bar{y}_{G1})^2 + \sum_{i=m+1}^N (y_{(i)} - \bar{y}_{G2})^2$$

(N4) Select as a candidate that partition which results in the maximum  $R_m$ . Let this be denoted as the  $M^{\text{th}}$  partition. Thus,

$$R_M = \max_{1 \leq m \leq N-1} R_m$$

(N5) Consider  $y_{(1)}, \dots, y_{(M)}$  as a random sample from  $N(\mu_1, \sigma^2)$  and  $y_{(M+1)}, \dots, y_{(N)}$  as a random sample from  $N(\mu_2, \sigma^2)$ . Choose a significance level  $\alpha$  and test the hypotheses:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

using the statistic  $R_M$ . The appropriate critical values given in Figure 2 were calculated by Engelman and Hartigan (1969).

(N6) If  $H_0$  is not rejected, conclude that there is no active factor. If  $H_0$  is rejected, define  $w_2, \dots, w_N$  such that

<u>N</u>	<u><math>\alpha = .05</math></u>	<u><math>\alpha = .25</math></u>
5	24.0	7.83
6	14.1	5.96
7	10.5	5.02
8	8.39	4.43
9	7.18	4.06

Figure 2: Critical Values of  $R_M$

$$w_1 = \begin{cases} 0 & \text{if } y_i \text{ is in the group of observations containing } y_1 \\ 1 & \text{if } y_i \text{ is not in the group of observations containing } y_1 \end{cases}$$

(N7) Follow steps D3 and D4 given previously.

It should be noted that for the deterministic case, the preceding steps yield results equivalent to those produced by steps D1 through D4. However, they are more cumbersome.

### III. PERFORMANCE EVALUATION

The central topic of this paper is evaluation of the performance of the revised Ott-Wehrfritz procedure in the nondeterministic case. In this case, of course, a factor may be identified as active when, in fact, all factors are inactive. The probability of this Type I error may be controlled by selection of the appropriate critical value.

On the other hand, if there is an active factor, the analysis may not identify this factor because of random error. Two different errors are possible. Either

(1) the value of the statistic is too small to conclude that there is an active factor,

or (2) the wrong factor is identified as active.

At this juncture, define the two events CD and A, where CD is the event that the correct decision is made and A is the event that there is an active factor. In this section, the performance of the revised Ott-Wehrfritz procedure is evaluated by examining, for  $\alpha = .05, .25$ , and 1.00, the value of  $P(CD|A)$ , the probability of correctly selecting the active factor when one exists.

It should be noted that if  $\alpha = 1.00$ , an implicit assumption is made that one active factor is definitely present. Thus, for  $\alpha = 1.00$  it will never be concluded that there is no active factor.

If there is one active factor, the observations  $y_1, \dots, y_N$  will include  $n$  observations from  $N(\Delta, \sigma^2)$  and  $N - n$  from  $N(-\Delta, \sigma^2)$ , assuming

without loss of generality that  $\beta_0 = 0$ . Of the  $2^{N-1} - 1$  possible factors to be screened in the  $N$  runs, the corresponding columns (based on the binary numbers) in the design matrix are such that  $C_n^{N-1}$  columns consist of  $n$  +1's and  $(N - n)$  -1's for  $n = 1, \dots, N - 1$ . If  $A_n$  denotes the event that the column corresponding to the active factor contains exactly  $n$  +1's, then the probability of interest may be written as:

$$\begin{aligned} P(CD|A) &= \sum_{n=1}^{N-1} P(CD|A_n)P(A_n) \\ &= \sum_{n=1}^{N-1} P(CD|A_n) \cdot [C_n^{N-1} / (2^{N-1} - 1)]. \end{aligned} \quad (1)$$

As might be expected,  $P(CD|A_n)$  is unwieldy to evaluate analytically since it represents the probability that for  $n$  observations from  $N(\Delta, \sigma^2)$  and  $N - n$  from  $N(-\Delta, \sigma^2)$ , the maximum BSS/WSS ratio is produced by the corresponding partition of the observations. Thus, Monte Carlo evaluation was used.

#### A. MONTE CARLO PROCEDURES

In each case considered, it is assumed that for an  $N$  run design a total of  $2^{N-1} - 1$  factors are to be examined. To evaluate  $P(CD|A)$ , the probability of interest, a Monte Carlo procedure was used to estimate  $P(CD|A_n)$  for  $n = 1, \dots, N - 1$ . In general, the procedure used randomly generated  $x_1, \dots, x_{N-n}$  from  $N(-\Delta, \sigma^2)$  and  $y_1, \dots, y_n$  from  $N(\Delta, \sigma^2)$ . The  $x$ 's were ordered as  $x_{(1)} < \dots < x_{(N-n)}$  and the  $y$ 's were ordered as  $y_{(1)} < \dots < y_{(n)}$ .

If  $x_{(N-n)} < y_{(1)}$ , the  $N - 1$  possible partitions of the ordered observations were evaluated to determine whether the two groups  $(x_1, \dots, x_{N-n})$

and  $(y_1, \dots, y_n)$  resulted in the maximum BSS/WSS ratio, and if so, whether the resulting value was greater than the upper .05 and .25 points of the null distribution. (If  $x_{(N-n)} > y_{(1)}$ , of course, the two groups of interest do not result in the maximum value of BSS/WSS.)

The resulting estimate of  $P(CD|A)$  was obtained by substituting the estimates of  $P(CD|A_n)$  into equation (1). It should be noted that, because of symmetry,  $P(CD|A_n) = P(CD|A_{N-n})$ . Therefore, this fact permitted estimation of  $P(CD|A)$  with a reduced total number of iterations. For each probability  $P(CD|A_n)$  estimated by the Monte Carlo procedure, 625 iterations were used. This resulted in estimated standard errors ranging between 0.000 and 0.014.

To provide a check on the Monte Carlo procedure,  $P(CD|A)$  was calculated by Monte Carlo for the (degenerate) case  $N = 3$ ,  $\Delta = 0$ ,  $\sigma = 1$ . In this case, it can be shown analytically using symmetry arguments that

$$P(CD|A) = P(CD|A_1) = P(CD|A_2) = 1/6.$$

Based on 1000 iterations, this probability was estimated to be .152, well within the 95% confidence interval of (.144, .190).

## B. RESULTS

The Monte Carlo procedure was used to obtain estimates of  $P(CD|A)$  for  $N = 5, 6, 7, 8, 9$  and  $\sigma = r|\Delta|$  for  $r = 0, .1, .2, .3, .4, .5$  where  $\Delta$  is the coefficient of the active factor. Figures 3, 4, and 5 present, for various values of  $\sigma$ , estimates of  $P(CD|A)$  corresponding to  $\alpha = .05, .25$ , and 1.00 respectively. As previously noted, the maximum estimated standard error for any table entry is 0.014.

<u>N</u>	<u>K</u>	<u>r = 0</u>	<u>r = .20</u>	<u>r = .40</u>	<u>r = .60</u>
5	15	1.000	0.763	0.223	0.092
6	31	1.000	0.919	0.311	0.119
7	63	1.000	0.969	0.392	0.128
8	127	1.000	0.988	0.488	0.121
9	255	1.000	0.994	0.567	0.169

Figure 3: Estimated Values of  $P(CD|A)$  Corresponding to  $\alpha = .05$  When  $\sigma = r|\Delta|$  and There Is One Active Factor with Coefficient  $\Delta$ .  
(Maximum Estimated Standard Error Is 0.013.)



<u>N</u>	<u>K</u>	<u>r = 0</u>	<u>r = .20</u>	<u>r = .40</u>	<u>r = .60</u>
5	15	1.000	0.994	0.650	0.320
6	31	1.000	0.998	0.703	0.372
7	63	1.000	0.999	0.763	0.365
8	127	1.000	0.999	0.812	0.358
9	255	1.000	1.000	0.828	0.401

Figure 4: Estimated Values of  $P(CD|A)$  Corresponding to  $\alpha = .25$  When  $\sigma = r|\Delta|$  and There Is One Active Factor with Coefficient  $\Delta$ . (Maximum Estimated Standard Error Is 0.014.)

<u>N</u>	<u>K</u>	<u>r = 0</u>	<u>r = .20</u>	<u>r = .40</u>	<u>r = .60</u>
5	15	1.000	1.000	0.944	0.693
6	31	1.000	1.000	0.927	0.669
7	63	1.000	1.000	0.914	0.626
8	127	1.000	1.000	0.918	0.587
9	255	1.000	1.000	0.916	0.583

Figure 5: Estimated Values of  $P(CD|A)$  Corresponding to  $\alpha = 1.00$  When  $\sigma = r|\Delta|$  and There Is One Active Factor with Coefficient  $\Delta$ . (Maximum Estimated Standard Error Is 0.014.)

Again it should be pointed out that the results in Figure 5 for  $\alpha = 1.00$  may be interpreted as the chances of identifying the correct factor when it is known that there is exactly one active factor present. In other words, since a critical value of zero corresponds to  $\alpha = 1.00$ , a correct decision is made if the active factor corresponds to the largest  $R_m$  ratio,  $R_M$ , regardless of its magnitude. From Figure 5, it can be seen that the probability that the active factor does correspond to  $R_M$  decreases as  $N$  increases, as intuition would suggest.

However, for the other values of  $\alpha$  considered, this probability must be multiplied by the probability that  $R_M$  is greater than the appropriate critical value. This latter probability, which increases with increasing  $N$ , offsets the decreasing nature of the former probability to produce, for  $\alpha = .05$  and  $\alpha = .25$ , the resulting probability values which also increase with increasing  $N$ .

#### IV. SUMMARY AND DISCUSSION

It must be emphasized that if the Ott-Wehrfritz procedure (or its revised version given here) is to be used, the assumption that there is at most one factor is critical. If this assumption is not true, many possible explanations of the data exist other than that provided by applying the procedure. Although this assumption may appear unrealistic, there may be situations where it is thought, a priori, that there is an extremely small probability (say less than  $1/3K$ ) that any given factor in the  $K$  factors is active. Then, the chances of encountering two or more active factors may be negligible.

Nonetheless, as Figures 3 through 5 show, the existence of even a relatively small amount of random error renders essentially useless a procedure which performs well in the deterministic case. In summary, the revised Ott-Wehrfritz procedure may provide a reasonable approach if both of the following conditions hold:

(1) There is at most one active factor

and (2)  $\sigma < .2\Delta$ , where  $\Delta$  is the coefficient of the active factor if there is one.

Otherwise, adopting this procedure would tend to cause more grief than benefit.

#### V. REFERENCES

Engelman, L. and Hartigan, J. A. (1969). "Percentage Points of a Test for Clusters," Journal of the American Statistical Association, 64, pp. 1647-1648.

Kleijnen, J. P. C. (1975). "Screening Designs for Poly-Factor Experimentation," Technometrics, 17, pp. 487-493.

Ott, E. R. and Wehrfritz, F. W. (1972). "A Special Screening Program for Many Treatments," Statistica Neerlandica, 26, pp. 165-170.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 113-1	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) PERFORMANCE EVALUATION OF A SPECIFIC FACTOR SCREENING TECHNIQUE		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Dennis E. Smith		8. CONTRACT OR GRANT NUMBER(s) N00014-79-C-0650 N00014-75-C-1054
9. PERFORMING ORGANIZATION NAME AND ADDRESS Desmatics, Inc. P. O. Box 618 State College, PA 16801		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NR 274-317 NR 042-334
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217		12. REPORT DATE March 1980
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Distribution of this report is unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Factor screening Experimental design		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  In many experimental situations (particularly in computer simulation studies) a large number of potentially important factors exist. Because of time and budget limitations, it is imperative to screen these factors in order to identify a subset which should be subjected to more detailed examination. This paper evaluates the performance of a factor-screening technique which has been proposed for use when it is known that there is at most one active factor (i.e., a factor which has an effect on the response of interest). -over-		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20 (continued)

Performance evaluation reveals that the existence of even a relatively small amount of random error renders essentially useless a procedure which performs well in the deterministic case.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)